



Analytical sensitivity analysis and topology optimization of nonlinear resonant structures with hardening and softening behavior

Dou, Suguang; Jensen, Jakob Søndergaard

Publication date:
2014

[Link back to DTU Orbit](#)

Citation (APA):

Dou, S., & Jensen, J. S. (2014). *Analytical sensitivity analysis and topology optimization of nonlinear resonant structures with hardening and softening behavior*. Abstract from 17th U.S. National Congress on Theoretical and Applied Mechanics, East Lansing, Michigan, United States.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Analytical sensitivity analysis and topology optimization of nonlinear resonant structures with hardening and softening behavior

Suguang Dou, Dept. Mechanical Eng., Technical University of Denmark, sudou@mek.dtu.dk
Jakob Jensen, Dept. Electrical Eng., Technical University of Denmark, json@elektro.dtu.dk

With the increasing applications of nonlinear resonant structures [1], researchers are interested in controlling the hardening and softening behavior [2]. We present a systematic procedure for topology optimization of nonlinear resonant structures with hardening and softening behavior. The combination of the finite element method (FEM) and alternating frequency/time domain method (AFT) is used to compute the nonlinear vibrational response. The finite element model facilitates an element-density-based parameterization for topology optimization [3] and the alternating frequency/time domain method takes advantage of the ease in evaluating complex nonlinearities in time domain [4].

Mechanical structures that possess geometrical nonlinearities are used to demonstrate the applicability and efficiency of the proposed procedure, and is here illustrated using 2D continuum structures. Two kinds of optimization problems are considered, including bandwidth problems and dynamic compliance problems. For bandwidth problems, the relative and absolute frequency shifts of resonance peaks caused by hardening and softening behavior are optimized. For dynamic compliance problems we consider point vibration amplitudes and total energy levels of the structure as optimization objectives. In order to speed up the computations we apply analytical sensitivity analysis [5] instead of the commonly applied finite difference approximation. For large FE models this leads to computational savings on up to several orders of magnitude. The optimization problem for relative frequency shift can be formulated as:

$$\begin{aligned} & \min_{x_e} \pm \gamma \\ \text{s.t. : } & \gamma = \frac{\omega^* - \omega_L}{\omega_L}, \quad \mathbf{K}_L \Phi = \omega_L^2 \mathbf{M} \Phi, \quad (\omega^*)^2 \bar{\mathbf{M}} \bar{\mathbf{q}} + (\omega^*) \bar{\mathbf{C}} \bar{\mathbf{q}} + \bar{\mathbf{g}} = \bar{\mathbf{f}}, \quad b_{i1} = 0, \\ & E_e = E_{\min} + (x_e)^p (E - E_{\min}), \quad \rho_e = \rho_{\min} + (x_e)^q (\rho - \rho_{\min}), \\ & \sum_{e=1}^{N_e} x_e \leq \alpha N_e, \quad 0 \leq x_e \leq 1. \end{aligned} \tag{1}$$

where ω^* is the nonlinear resonant frequency at the specified load, ω_L is the linear resonant frequency, γ measures the extent of hardening and softening behavior, E_e and ρ_e are interpolated Young's modulus and mass density of each element, p and q are integers with typical values 3 and 1, $x_e (e = 1, \dots, N_e)$ are design variables and α represents a maximum allowable volume fraction. Note that b_{i1} is the coefficient of one sinusoidal term and the condition $b_{i1} = 0$ is related to the phase lag quadrature criterion [6]. The barred matrices and vectors are in the frequency domain and can be found in [5].

Example Consider a thin 2D continuum structure in micro-scale with doubly-clamped boundary condition. An in-plane vertical time-harmonic load is applied at the center node of the structure. The material is Silicon nitride and its mechanical properties are: Young's modulus $E = 241$ GPa, Poisson's ratio $\nu = 0.23$ and mass density 300 kg/m^3 . Plane stress conditions are assumed and preliminary results are obtained

with a coarse discretization of 40×20 elements. The initial design is simply the uniform distribution of material in the design domain. The specified load is so small that the relative frequency shift around the first flexural mode is less than 0.04% for the initial design. The objective function is to maximize the relative frequency shift caused by hardening resonance around the first flexural mode. In the optimized design, the solid structure (black area) is not allowed to fill more than half of the total area of the design domain. The second optimized design is obtained by imposing an additional constraint on minimal eigenvalue in order to ensure sufficient structural rigidity.

For both optimized designs shown in Figure 1, the relative frequency shifts are more than 34%. A physical interpretation of the optimized structures is that the midplane stretching plays a key role when four beams are connected to one central mass and two boundaries through eight hinge-like connections. In the designs point-hinges appear as well as gray elements that correspond to intermediate material properties. Further work is currently being undertaken to improve the designs by employing a finer mesh, projection filtering and minimal length control techniques. However, in reality the gray elements may be realized using a weaker material such as polymer. For example, polymeric hinges have been used in the design of micro-mirrors to achieve large deflection at small external forces.

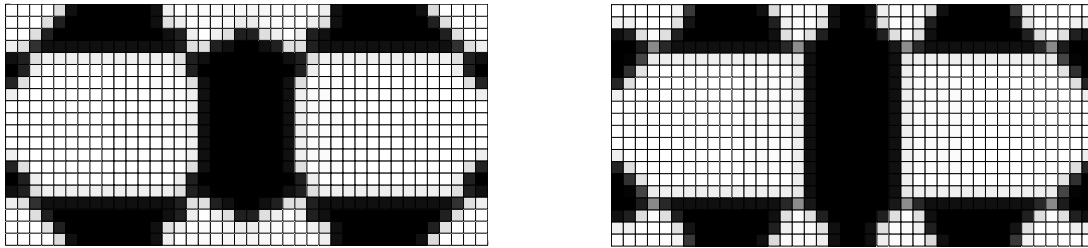


Figure 1. Optimized design with constraint on maximal volume (left) and an additional constraint on minimal eigenvalue (right)

Conclusion A systematic procedure is proposed for optimization of nonlinear resonant structures with hardening and softening behavior by tailoring the geometric nonlinearity. The combination of analytical sensitivity analysis and the topology optimization procedure is shown to be a promising tool in design of nonlinear micro- and nano-resonators.

References

- [1] Jeffrey F. Rhoads, Steven W. Shaw, and Kimberly L. Turner. Nonlinear dynamics and its applications in micro- and nanoresonators. *Journal of Dynamic Systems, Measurement, and Control*, 132(3):034001, 2010.
- [2] Hanna Cho, Bongwon Jeong, Min-Feng Yu, Alexander F. Vakakis, D. Michael McFarland, and Lawrence A. Bergman. Nonlinear hardening and softening resonances in micromechanical cantilever-nanotube systems originated from nanoscale geometric nonlinearities. *International Journal of Solids and Structures*, 49(15–16):2059 – 2065, 2012.
- [3] Martin P. Bendsøe and Ole Sigmund. *Topology Optimization: Theory, Methods and Applications*. Springer, 2003.
- [4] T. M. Cameron and J. H. Griffin. An alternating frequency/time domain method for calculating the steady-state response of nonlinear dynamic systems. *Journal of Applied Mechanics*, 56(1):149–154, March 1989.
- [5] Suguang Dou and Jakob S. Jensen. Optimization of nonlinear structural resonance using the incremental harmonic balance method. 2013. Submitted.
- [6] M. Peeters, G. Kerschen, and J.C. Golinval. Modal testing of nonlinear vibrating structures based on nonlinear normal modes: Experimental demonstration. *Mechanical Systems and Signal Processing*, 25(4):1227 – 1247, 2011.